PHYS 100C, LECTURE 12

Thursday, May 07, 2009
2:01 PM

$$\vec{E}(r,t) = \frac{q}{4\pi\epsilon_0}\cdot\frac{r}{(\hat{r}\cdot u)^3}\left[(c^2-v^2)u + \hat{r}\times(u\times a)\right]$$

velocity field; acc. field

$$\vec{B} = \frac{\hat{r}\times\vec{E}}{c}$$

$$S = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) = \frac{\vec{E}^2\hat{r} - (\hat{r}\cdot\vec{E})\vec{E}}{\mu_0 c}$$

once again, E has

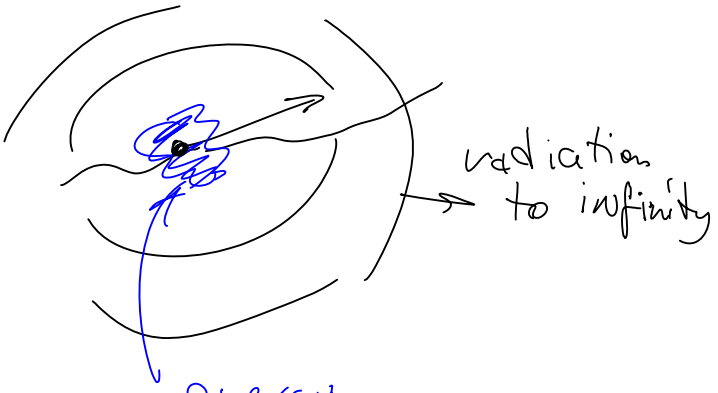
two terms; velocity field $\sim 1/r^2$
accelerator field $\sim 1/r$

For S:

accel. term^2 $\rightarrow \frac{1}{r^2}$ } radiation

velocity term^2 $\rightarrow \frac{1}{r^4}$ } "cloud" of energy

CROSS-term $\rightarrow \frac{1}{r^3}$ } "cloud" of energy



energy follows particle like flies on garbage truck

Reaction force?

Larmor Law:

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

($\langle S \rangle$ integrated over $4\pi R^2$)

$$F \cdot v = -P = -\frac{\mu_0 q^2 a^2}{6\pi c}$$

(Power is force x velocity)

True ONLY on average!
WRONG FOR INSTANTENEOUS F.

Why? Neglects energy exchange with "entourage" energy cloud of garbage flies.

IF we consider times t_1 and t_2 that have the same a and v, then $S(t_1) = S(t_2)$ and "cloud" energy is the same, so:

$$\int_{t_1}^{t_2} F \cdot v \, dt = -\frac{\mu_0 q^2}{6\pi c} \int_{t_1}^{t_2} a^2 \, dt$$

(reaction force due to energy radiated out).

but $a^2 = \left(\frac{\partial v}{\partial t}\right) \cdot \left(\frac{\partial v}{\partial t}\right) = \left(v \cdot \frac{\partial v}{\partial t}\right)' - v \frac{\partial^2 v}{\partial t^2}$

$$\int_{t_1}^{t_2} a^2 \, dt = v \frac{\partial v}{\partial t}\Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} v \frac{\partial^2 v}{\partial t^2}$$

$= 0$ since $v(t_2) = v(t_1)$
$a(t_2) = a(t_1)$

t_2

$$\int_{t_1}^{t_2}\left(\vec{F}\cdot\vec{v} - \frac{\mu_0 q^2}{6\pi c}\dot{\vec{a}}\cdot\vec{v}\right)\cdot dt = 0$$

which is true if

$$\vec{F} = \frac{\mu_0 q^2}{6\pi c}\dot{\vec{a}}$$

jerk (just like Steve Martin movie)

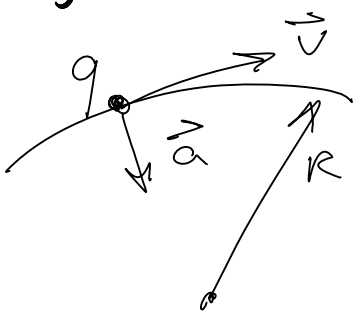
What does it mean?

Energy exchange w/ cloud is such that $\vec{F} \sim \dot{\vec{a}}$

E.g. if $a = 0$ (no radiation) but $\dot{a} \neq 0$ there's force (creating or destroying "cloud").

Weird stuff! (see textbook for more weirdness).

* Synchrotrons



Particle q moving on a circle R

Classical $a = \frac{v^2}{R}$

Relativistic:

$$a = \frac{1}{m}\cdot\frac{\partial p}{\partial \tau} = \frac{1}{m}\frac{\partial(\gamma m v)}{\partial t/\gamma} = \gamma^2\frac{\partial v}{\partial t} = \gamma^2\frac{v^2}{R}$$

$\partial\tau = \frac{\partial t}{\gamma}$ ← time contraction

$P \sim a^2$ (power radiated)

$P \sim \gamma^4 \frac{v^4}{R^2}$

Advanced Photon Source, Chicago IL

Energy of electrons 7 GeV
rest mass of e^- 0.5 MeV

$\gamma = \frac{7\,GeV}{0.5\,MeV} = 14{,}000$ (!)

Radiation is a problem for high energy physics.

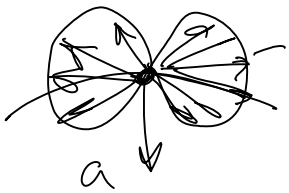
LHC aims to ramp up from 450 GeV to 7 TeV
(they use protons, not electrons)

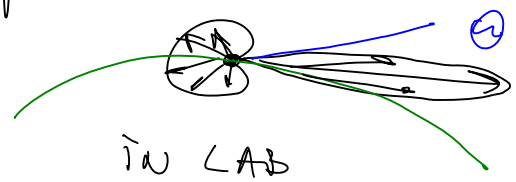
Increase energy by $\times 10$ → power bill goes up by $\times 10{,}000$!

You can reduce it by making R large (LHC is 27 km circumference).

Can we make it 2700 km to cut power BIL By $\times 10{,}000$?
$\left(P \sim \frac{1}{R^2}\right)$

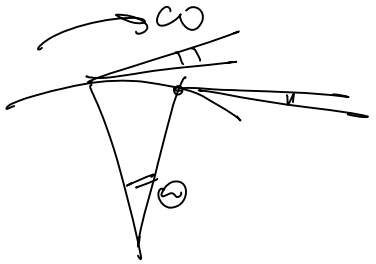
Energy spectrum and pulse duration:





IN LAB FRAME

"SEARCHLIGHT" OF ANGLE $\Theta \sim 1/\gamma$



see the light for time:

$$\Delta t = \frac{\Theta}{\omega} \cdot \left(1 - \frac{v}{c}\right)$$

Note: $1 - v/c = \frac{1 - v^2/c^2}{1 + v/c} \approx \frac{1}{2\gamma^2}$

Since $\Theta = \frac{1}{\gamma}$,

$$\Delta t = \frac{1}{2\gamma^3 \omega} \Rightarrow$$ Fourier components of short pulse

$$\omega_{MAX} \sim \frac{2\pi}{\Delta t} = 4\pi\gamma^3\omega$$

$$\lambda_{min} \sim \frac{2\pi c}{\omega_{max}} = c\Delta t = \frac{c}{2\gamma^3\omega}$$

But $\omega = \frac{v}{R} = \frac{c}{R}$ $(v \approx c)$

$$\lambda_{min} \sim \frac{R}{2\gamma^3} \approx \frac{1{,}000\,m}{2 \cdot (10^4)^3} =$$

$$= 5\,\text{Å} \quad (\text{X-RAYS!})$$

* Next week: GUEST SPEAKER, PROF. SUNIL "SUNNY" SINHA (TUESDAY), MIDTERM (THUR.)